Saturation Numbers of Graphs

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Typical problems in extremal graph theory ask for the maximum or minimum value of parameters of graphs with certain properties. A classical problem falling within this framework, posed by Turán in 1941, asks for the maximum number of edges in an F-free graph on n vertices. Recall that a graph H is F-free if H has no a subgraph isomorphic to F. A dual problem, posed by Zykov in 1949, asks for the minimum number of edges of an F-saturated graph on n vertices. Recall that a graph H is F-saturated if H is F-free and adding any missing edge to H creates a copy of F. A dynamic version of the saturation problem, posed by Bollobás in 1968, asks for the minimum number of edges of a weakly F-saturated graph on n vertices. Recall that a graph H is weakly F-saturated if H is F-free and all missing edges in H can be added to H in some order so that a new copy of F is created at each step.

In this talk, we briefly review some known results and open conjectures on saturation and weak saturation numbers of graphs and random graphs.