Institute for Research in Fundamental Sciences
School of Mathematics-Isfahan Branch

Seminar on

Representation Theory of Artin Algebras

October 19-20, 2016

Organizers:

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Its goal was to carry out original research in Physics and Mathematics. Schools of Mathematics and Physics were the first two schools of IPM. Now it has ten schools conducting research in many different areas of fundamental sciences. Although the main activities of IPM are based on researchers resided in Tehran, it has some programs for supporting people from around the country.

School of Mathematics of IPM, started its work by three core research groups. Now, it has three research groups in Combinatorics and Computing, Commutative Algebra and Logic, some resident researchers and also non-resident researchers.

The first branch of the School of Mathematics of IPM has been founded in September 22, 2012 in Isfahan.

The goal of this branch is not only providing a good atmosphere for the researchers living in Isfahan and cities nearby, but also acting as a complementary to the School of Mathematics in Tehran for supporting the non-resident researchers across the country.

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Seminar Program

o 1st day, Wednesday October 19:

Time	Speaker	Title of the Talk
9:00-10:00	**C.M. Ringel,** University of Bielefeld, Germany	The n-Kronecker Modules I
10:10-11:10	**Rassol Hafezi,** IPM, Iran	Some New Variations Of Auslander's Formula And Applications
11:10-11:30	**Coffee Break**	
11:30-12:30	**Esmaeil Hosseini,** Shahid Chamran University of Ahvaz, Iran	On Pure Derived Categories Of Tensor Categories
12:30-14:00	**Lunch and Group Photo**	
14:00-15:00	**Hossein Eshraghi,** University of Kashan, Iran	Representation Dimension And Tilting Theory
15:00-15:20	**Coffee Break**	
15:20-15:50	**Payam Bahiraei,** IPM, Iran	Model Structures On The Category Of Complexes Of Quiver Representations
16:00-16:30	**Ali Mahin-Fallah,** IPM, Iran	On The Auslander-Reiten Conjecture For Algebras

Seminar Program

o 2nd day, Thursday October 20:

Time	Speaker	Title of the Talk
9:00-10:00	**C.M. Ringel,** University of Bielefeld, Germany	The n-Kronecker Modules II
10:10-11:10	**Shokrollah Salarian,** University of Isfahan and IPM, Iran	Modules Of Finite Cohen-Macaulay Type
11:10-11:30	**Coffee Break**	
11:30-12:30	**Alireza Nasr-Isfahani,** University of Isfahan and IPM, Iran	Strongness Of Companion Bases For Cluster-Tilted Algebras Of Finite Type
12:30-14:00	**Lunch**	
14:00-15:00	**Razieh Vahed,** IPM, Iran	Induced G-precoverings Of Triangulated Categories
15:00-15:20	**Coffee Break**	
15:20-15:50	**Ehsan Hakimian,** University of Isfahan, Iran	Annihilation Of Cohomology, Generation Of Modules And Finiteness Of Derived Dimension
16:00-16:30	**M. H. Keshavarz,** University of Isfahan, Iran	Homological Aspects Of Categorical Algebras (Special Bound Quiver Algebras)

The n-Kronecker Modules

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The n-Kronecker modules are the representations of the n-Kronecker quiver; this is the quiver with two vertices, namely a sink and a source, and n arrows. The n-Kronecker modules have to be considered as basic objects in mathematics. As an introduction for the two lectures, we will discuss the relevance of the n-Kronecker modules in representation theory and outline essential features. We will recall that the case n = 2 has been studied a long time ago in various disguises: by Weierstrass and Kronecker, by Hilbert and Grothendieck, and by many other mathematicians; this is the prototype of a tame module category. But not much is known for n $\geq$ 3.

The main part of the first lecture will be devoted to the role of bristles: these are the indecomposable modules of length 2. As we will show there is an abundance of n-Kronecker modules which are generated by bristles.

In the second lecture we will determine the elementary 3-Kronecker modules. Let us recall that a regular representation of a quiver is said to be elementary provided it is non-zero and not a proper extension of regular representations. Of course any regular representation has a filtration whose factors are elementary. It turns out that the elementary 3-Kronecker modules are either tree modules or circle modules, thus determined by combinatorial invariants and at most one scalar.

Some New Variations of Auslander's Formula and Applications

Rasool Hafezi
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Let $\mathcal{C}$ be an abelian category. A contravariant functor F from $\mathcal{C}$ to the category of abelian groups $\mathcal{A}b$ is called finitely presented, or coherent [A], if there exists an exact sequence

$$\mathrm{Hom}_{\mathcal{C}}(-,X) \longrightarrow \mathrm{Hom}_{\mathcal{C}}(-,Y) \longrightarrow F \longrightarrow 0$$

of functors. Let mod$\mathcal{C}$ denote the category of all coherent functors. The systematic study of mod$\mathcal{C}$ is initiated by Auslander [A]. He, not only showed that mod$\mathcal{C}$ is an abelian category of global dimension less than or equal to two but also provided a nice connection between mod$\mathcal{C}$ and $\mathcal{C}$. This connection, which is known as Auslander's formula [L, K], suggests that one way of studying $\mathcal{C}$ is to study mod$\mathcal{C}$, that has nicer homological properties than $\mathcal{C}$, and then translate the results back to $\mathcal{C}$. In particular if we let $\mathcal{C}$ to be modΛ, where Λ is an artin algebra, Auslander's formula translates to the equivalence

$$\frac{\mathrm{mod}(\mathrm{mod}\Lambda)}{\{F \mid F(\Lambda)=0\}} \simeq \mathrm{mod}\Lambda$$

of abelian categories. As it is mentioned in [L], 'a considerable part of Auslander's work on the representation theory of finite dimensional, or more general artin, algebras can be connected to this formula'.

Recently, Krause [K] established a derived version of this equivalence. In my talk, some different (relative and derived) versions of this formula will be explained. Then I will give some applications of our results for artin algebras.

References

[A] M. Auslander, *Coherent functors,* 1966 Proc. Conf. Categorical Algebra (La Jolla, Calif., 1965) pp. 189-231 Springer, New York.

[L] H. Lenzing, *Auslander's work on Artin algebras,* Algebras and modules, I (Trondheim, 1996), 83-105, CMS Conf. Proc., 23, Amer. Math. Soc., Providence, RI, 1998.

[K] H. Krause, *Deriving Auslander's formula,* Doc. Math. **20** (2015) 669-688.

On Pure Derived Categories of Tensor Categories

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Let $\mathcal{G}$ be a locally finitely presented tensor category. There are two different pure exact structure on the category $\mathbf{C}(\mathcal{G})$ of all complexes in $\mathcal{G}$, the categorical purity and the tensor purity. Recently, categorical pure derived categories of $\mathcal{G}$ have been studied in more details. In this talk, we describe the difference between those purities and show that tensor pure derived categories of $\mathcal{G}$ have suitable replacements. In addition, we talk about a problem which is posed by Krause and investigate a relation between tensor pure derived categories and Grothendieck Duality Theorem.

Representation Dimension and Tilting Theory

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Abstract. Let Λ be an artin algebra over a commutative artinian ring R and let T be a tilting Λ-module with endomorphism $\Gamma = \mathrm{End}_\Lambda(T)$. In this talk, we will study the representation dimension of Γ. Our approach uses the methods of classical tilting theory and the main goal is to obtain some upper bound on rep.dim(Γ). Firstly, a very brief overview of the main topics of classical tilting theory will be presented and we will proceed by focusing on algebras which are Gorenstein and of finite Cohen-Macaulay type and the tilting modules which are simultaneously separating and splitting. The attempt lies in the direction to outline the main steps towards the proof of the following result: for an integer $n \geq 1$, if Λ is n-Gorenstein of finite Cohen-Macaulay type and T is a proper separating splitting tilting module, then rep.dim(Γ) $\leq n+2$. The upshot is that if Λ is a n-Gorenstein artin algebra of finite Cohen-Macaulay type admitting a proper separating-splitting tilting module, then rep.dim(Λ) $\leq n+2$.

Model Structures on The Category of Complexes of Quiver Representations

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In this talk, we study the category $\mathbb{C}(\mathrm{Rep}(\mathcal{Q},\mathcal{G}))$ of complexes of representations of quiver $\mathcal{Q}$ with values in a Grothendieck category $\mathcal{G}$. We develop a method for constructing some model structures on $\mathbb{C}(\mathrm{Rep}(\mathcal{Q},\mathcal{G}))$ based on componentwise notion. As an application of these model structure we introduce some descriptions of the derived category of complexes of representations of $\mathcal{Q}$ in Mod-R. We also study the morphism category $\mathbf{H}(R)$ and its two full subcategories, monomorphism category $\mathbf{S}(R)$ and epimorphism category $\mathbf{F}(R)$. We show that the well know equivalence between $\mathbf{S}(R)$ and $\mathbf{F}(R)$ can be extended to an auto-equivalence of $\mathbf{H}(R)$.

On The Auslander-Reiten Conjecture for Algebras

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A recent result of Huneke, Leuschke and Araya, asserts that if the Auslander-Reiten conjecture holds in codimension one for a commutative Gorenstein ring R, then it holds for R. We extend this result to left Gorenstein R-algebra Λ, whenever R is a commutative Gorenstein ring. This, in particular, implies that any finitely generated self-orthogonal Gorenstein projective Λ-module is projective, provided Λ is an isolated singularity and $\dim R \geqslant 2$.

Modules of Finite Cohen-Macaulay Type

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Assume that (R, m) is a d-dimensional commutative noetherian complete Cohen-Macaulay local ring. In this talk we investigate modules of finite Cohen-Macaulay type. An R-module M is said to be of finite Cohen-Macaulay type, if it is the direct sum of (arbitrarily many) copies of a finite number, up to isomorphisms, indecomposable maximal Cohen-Macaulay modules.

Strongness of Companion Bases for Cluster-Tilted Algebras of Finite Type

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For every cluster-tilted algebra of simply-laced Dynkin type we provide a companion basis which is strong, i.e. gives the set of dimension vectors of the finitely generated indecomposable modules for the cluster-tilted algebra. This shows in particular that every companion basis of a cluster-tilted algebra of simply-laced Dynkin type is strong. Thus we give a proof of Parsons's conjecture. This talk is based on a joint work with K. Baur.

Induced G-precoverings of Triangulated Categories

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Covering techniques in representation theory have become important after the work of Bongartz-Gabriel [BG], Gabriel [G] and Riedtmann [Ri]. In fact, at first Riedtmann [Ri] introduce coverings of the Auslander-Reiten quiver Γ_Λ of a representation-finite algebra Λ. Bongartz and Gabriel [BG] developed this notion to provide concrete algorithms which enable us to construct the Auslander-Reiten quivers for plenty of algebras.

Let $\mathbb{k}$ be a field and G be a group. In [G] Gabriel introduced the notion of Galois G-covering of locally bounded $\mathbb{k}$-categories with a G-action, to present a technique for the computation of the indecomposable modules over a representation-finite algebra.

Locally bounded G-categories have been well investigated in connection with a so-called covering technique in representation theory of algebras, see [G]. The orbit category $\mathcal{C}/G$ and the canonical functor $P : \mathcal{C} \longrightarrow \mathcal{C}/G$ are naturally constructed from these data, and one studies relationships between Mod-$\mathcal{C}$ and Mod-$(\mathcal{C}/G)$.

Asashiba in [As] generalized the covering technique for an arbitrary $\mathbb{k}$-categories with a G-action to apply covering techniques to usual additive categories such as the homotopy category $\mathbb{K}(\text{Prj-}\mathcal{C})$ of projectives and he showed that the pushdown functor $P_{\bullet} : \mathbb{K}^{\mathrm{b}}(\text{prj-}\mathcal{C}) \longrightarrow \mathbb{K}^{\mathrm{b}}(\text{prj-}(\mathcal{C}/G))$ is a G-precovering.

Using this generalization, we intend to introduce G-precovering of bounded derived categories, singularity categories and Gorenstein defect categories which are induced by the pushdown functor $P_{\bullet}$. Moreover, we present some applications of our results.

This talk is based on a joint work with H. Asashiba and R. Hafezi.

References

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[Ri] C. Riedtmann, *Algebren, Darstellungskocher, Uberlagerungen und zuruck,* Comment. Math. Helv. **55** (1980) 199-224.

Annihilation of Cohomology, Generation of Modules and Finiteness of Derived Dimension

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In this talk we briefly review some basic concepts of uniform annihilation of cohomology and finiteness of derived dimension of module-finite Λ. Also close link between these two concepts will be revealed. In addition, we present situations that guarantee the existence of a uniform annihilation of cohomology as well as situations in which this property can be transferred between the base ring and its completion. We also show that when $(R; m)$ is a commutative Noetherian local ring, the R-modules locally free on the punctured spectrum are constructed from syzygies of finite length modules by taking direct sums/summands and d extensions.

Homological Aspects of Categorical Algebras
(Special Bound Quiver Algebras)

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Let R be a ring and $\mathcal{Q}$ be a finite quiver. We gave an explicit formula for the injective envelopes and projective precovers in the category $\mathrm{Rep}(\mathcal{Q}_I, R)$ of bound representations of $\mathcal{Q}$ by left R-modules, where I is a combination of monomial and commutativity relations. Some applications would be provided. In particular, it was shown that if R is an Iwanaga-Gorenstein ring, then so are these bound quiver algebras.

We also extended our formula to all terms of the minimal injective resolution of $R\mathcal{Q}$. Using such descriptions, we studied the Auslander-Gorenstein property of path algebras. In particular, we proved that the path algebra $R\mathcal{Q}$ is k-Gorenstein if and only if $\mathcal{Q} = \overrightarrow{A_n}$ and R is a k-Gorenstein ring, where n is the number of vertices of $\mathcal{Q}$.

In fact we studied some homological aspects of path algebras. Also we studied some homological Aspects of $U_N(R)$.

Note that the classical representation theory of quivers considers finite quivers and assume that the base ring is algebraically closed field and that all vector spaces involve are finite dimensional. But we wrote our results in greater generality (not just fileds) for some reasons: One of the advantage of working in the category of representations, with value in the category of R-modules when R is an arbitrary ring (not only field) is to study the category of representations of a quiver $\mathcal{Q}$ with relations over a field. On the other hand by working with them one can reproof and extend some interesting results in the literature.

IPM-Isfahan

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