Numerical Analysis with Kernels

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Abstract: Kernels are very useful tools in various fields of Numerical Analysis, including approximation, interpolation, meshless methods for solving partial differential equations, neural networks, surface construction, image analysis, and Machine Learning. This talk starts from the close connection of kernels to Hilbert spaces via either point evaluations, series expansions, feature maps, or Fourier transforms, leading to the most important examples of kernels. Then it is shown in quite some generality that using kernels leads to error-optimal numerical methods in Hilbert spaces of functions. This is the main reason why kernels are important for Numerical Analysis, and it will be illustrated by various examples ranging from numerical integration and differentiation to pointwise recovery of solutions of operator equations.

Surprisingly, kernel methods in simple implementations have a very close connection of stability to the approximation quality. If one tries to decrease errors, stability gets worse, and this can be proven to be unavoidable. But better choices of bases can overcome this dilemma, and some recent developments for increasing stability will be surveyed.

For various applications, users need new kernels with special properties, and it is shown how certain specific construction techniques yield new useful kernels.

During the talk, a few open problems will also be mentioned.

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