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A GENERALIZATION OF GROTHENDIECK'S VANISHING THEOREM

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ABSTRACT. Let X be a quasi-compact topological space, $\mathfrak{Mod}X$ be the category of all sheaves of abelian groups on X and $\text{Flat}X$ be its full subcategory of flat sheaves. If every object of $\text{Flat}X$ has finite cotorsion dimension, then we prove that for every $\mathcal{F} \in \text{Flat}X$, $\mathcal{G} \in \mathfrak{Mod}X$, and $i \geq n$, $\text{Ext}^i(\mathcal{F}, \mathcal{G}) = 0$, in particular, the cohomological group $H^i(X, \mathcal{G}) = 0$. If X is a coherent n -perfect (not necessarily of finite krull dimension) locally ringed space, we prove that every flat sheaf has finite pure injective dimension. Also, we show that if X is a scheme then there is an equivalence $\mathbf{K}(\text{Pinf}X) \rightarrow \mathbf{D}_{\text{pure}}(\text{Flat}X)$ of homotopy categories, whenever $\mathbf{K}(\text{Pinf}X)$ is the homotopy category of pure injective flat quasi-coherent sheaves and $\mathbf{D}_{\text{pure}}(\text{Flat}X)$ is the pure derived category of flat quasi-coherent sheaves.

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