

Nonlinear Eigenvalue Problems General Theory, Results and Methods

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A wide variety of applications requires the solution of a nonlinear eigenvalue problem

$$T(\lambda)x = 0$$

where $T(\lambda)$ is a family of linear operators on a Hilbert space depending on a complex parameter $\lambda \in D$. As in the linear case λ is called an eigenvalue of $T(\cdot)$ if the equation $T(\lambda)x = 0$ has a nontrivial solution $x \neq 0$, and then x is called a corresponding eigenelement.

Quadratic problems $T(\lambda) := \lambda^2 M + \lambda C + K$ arise in the dynamic analysis of structures, or vibrations of spinning structures yielding conservative gyroscopic systems, constrained least squares problems, and control of linear mechanical systems with a quadratic cost functional. Polynomial eigenvalue problems of higher degree than two arise when discretising a linear eigenproblem by dynamic elements or by least squares elements, and in the study of corner singularities in anisotropic elastic materials. Rational eigenproblems govern free vibrations of fluid solid structures or describe the electronic states of semi-conductor hetero-structures when considering an electron effective mass depending on the energy state. Finally, a more general dependence on the eigenparameter appears in vibrations of poroelastic and piezoelectric structures and in the stability analysis of vibrating systems under state delay feedback control.

In this talk we discuss the differences and commonalities of linear and nonlinear eigenvalue problems, we review approximation properties of one- and two-sided Rayleigh functionals, and present numerical methods for dense nonlinear eigenvalue problems based on linearization, the characteristic equation $\chi(\lambda) = \det T(\lambda)$, and Newton's method for determining simple eigenvalues and invariant pairs.

References

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