LOCATING AND COMPUTING THE EIGENVALUES OF MATRIX POLYNOMIALS USING TROPICAL ALGEBRA

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The known results of Hadamard, Ostrowski and Pólya for locating the roots of polynomials can be restated in terms of tropical algebra by using the tropical roots. These tropical roots are the points of non-differentiability of the tropical function $\mathsf{t}_{\times}p(x) = \max_{0\leq j\leq \ell} |a_j|x^j$ and can be computed in only $O(\ell)$ operations. We extend all these classical bounds to the case of the matrix polynomial $P(\lambda) = \sum_{j=0}^{\ell} \lambda^j A_j$ to locate its eigenvalues. Here we consider the tropical polynomial $\mathsf{t}_{\times}p(x) = \max_{0\leq i\leq \ell} \|A_i\|x^i$ by taking the norms of the coefficients of $P(\lambda)$. Our results yield conditions under which tropical roots offer order of magnitude approximations to the moduli of the eigenvalues of $P(\lambda)$.

We use these theoretical results to determine effective initial approximations for the numerical computation of the eigenvalues of matrix polynomials by means of simultaneous iterations, like the Ehrlich-Aberth method. Our numerical experiments show the computational advantage of these results.

We also develop a general scaling technique based on tropical algebra for the numerical computation of the eigenvalues of $P(\lambda)$ and we study the effect of this scaling on (a) the conditioning of linearizations of tropically scaled $P(\lambda)$ and (b) the backward stability of eigensolvers based on linearizations of tropically scaled $P(\lambda)$. We anticipate that the tropical roots of $\mathsf{t}_{\times}p(x)$, on which the tropical scalings are based, will help designing polynomial eigensolvers with better numerical properties than standard algorithms for polynomial eigenvalue problems such as that implemented in the MATLAB function `polyeig`.

References

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