Introduction to Group Actions

1st Problem Set
Due Mehr 25th, 1398

- Exercise 1.1.2, page 3 from [BS].
- Exercises 1.2.1, 1.2.2, 1.2.3 and 1.2.4, page 5 from [BS].
- Exercise 1.3.1, page 7 from [BS].
- Exercise 1.4.4, page 9 from [BS].
- Following the notation of [BS], let $\sigma : \Sigma_m^+ \to \Sigma_m^+$ be the full one-sided shift on infinite words of m symbols. Prove that this system has uncountably many points with dense orbits. (The same holds for the full two-sided shift.)
- Prove that for any natural number m, there exists a power n so that (σ, Σ_m^+) is a factor of (σ^n, Σ_2^+), where $\sigma^n = \underbrace{\sigma \circ \cdots \circ \sigma}_{n}$.

  (For definition of "factor", see page 3, section 1.1 in [BS].)
- Denote the rotation of angle α on the unit circle (S^1) by R_α.
  a) Prove that for any α, the rotation (R_α, S^1) is not a factor of the shift map (σ, Σ_2^+).
  b) Prove that for any α, the shift map (σ, Σ_2^+) is not a factor of the rotation (R_α, S^1).
- It is true that closure of any orbit for $\sigma : \Sigma_m^+ \to \Sigma_m^+$ contains a periodic point?

References

[BS] Brin, M., Stuck, G., *Introduction to Dynamical Systems*, Cambridge University Press.

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